Explosion in underground cavity

(E. Fermi in R.L. Garwin's notebook LANB 3616)

Total energy $= 5\times10^{21}$ ergs $= W$

Initial radius $R = 33$ m

Initial volume $\frac{4\pi}{3}R^3 = 1.25\times10^5$ m^3

$$p = \frac{W}{V}(\gamma-1) = \frac{5\times10^{21}}{1.25\times10^{11}}\,\frac{2}{3} = 2.7\times10^{10}$$

From p. 6

Assume equation of state of rock

$$E = \frac{1}{2}k\,(v_0 - v)^2 = \text{en. per cc}$$

$$p = (v_0 - v)\,k$$

$$c = \sqrt{k\,v_0^2}$$

$v_0 = .4 \quad c = 5\times10^5 \quad k = 1.57\times10^{12}$

From 3rd Hugoniot

$$\frac{1}{2}k\,(v_0 - v_1)^{\cancel{2}} = \frac{1}{2}p\,\cancel{(v_0 - v_1)}$$

$$v_0 - v_1 = \frac{p}{k} = \frac{2.7\times10^{10}}{15.7\times10^{11}} = .0172$$

$v_0 = .4000$

$v_1 = .3828$

"Explosion in underground cavity," seven pages by Enrico Fermi in R.L. Garwin's Los Alamos notebook (LANB 3616), calculating the radiated wave from an explosion in an underground cavity of initial radius 33 m, and total energy 100 kt. (070050.EF)

From 2nd Hug

$$U^2 = \frac{v_0^2}{v_0 - v_1}\, p = \frac{.4^2}{.0172}\, 2.7 \times 10^{10} = 25.1 \times 10^{10}$$

From 1st Hug

$$u = \frac{v_0 - v_1}{v_0}\, U = \frac{.0172}{.4}\, 5 \times 10^5 = 2.15 \times 10^4$$

radial expansion $q' - 1$ $\quad$ q = new radius

lateral " $\frac{q}{r} - 1$

Density of elastic energy

$$\frac{\alpha}{2}\left[(q'-1)^2 + 2\left(\frac{q}{r}-1\right)^2\right] + \beta\left[\left(\frac{q}{r}-1\right)^2 + 2\left(\frac{q}{r}-1\right)(q'-1)\right]$$

Elastic energy =

$$W_{el} = \int 4\pi r^2\, dr \left\{ \qquad \right\}$$

Minimum problem

$$r^2 q'' + 2r q' - 2q = 0$$

Solution

$$q = r + \frac{a}{r^2}$$

$$\frac{q}{r} - 1 = \frac{a}{r^3} \qquad q' - 1 = -\frac{2a}{r^3}$$

$$W_{el} = 4\pi(\alpha - \beta)\frac{a^2}{r_0^3}$$

$$p_{el} = 2(\alpha - \beta)\frac{a}{r_0^3} = 2(\alpha-\beta)\frac{q_0 - r_0}{r_0}$$

$$\frac{\alpha}{2} + \beta = \frac{3}{2} k v_0^2$$

$$\sigma = \frac{\beta}{\alpha + \beta} = \text{Poisson ratio} = .3$$

$$.7\beta = .3\alpha \qquad \alpha = \frac{7}{3}\beta$$

$$\left(\frac{7}{6} + 1\right)\beta = \frac{3}{2}\, 1.57 \times 10^{12} \times .16 = .378 \times 10^{12}$$

$$\beta = 1.74 \times 10^{11} \qquad \alpha = 4.06 \times 10^{11} \qquad \alpha - \beta = 2.32 \times 10^{11}$$

Adiabatic gas expansion

$$p\left(r^3\right)^{5/3} = \text{const}$$

$$p r^5 = 2.7\times 10^{10} \times 3300^5 = 1.1\times 10^{28}$$

$$2(\alpha-\beta)\frac{q_0 - r_0}{r_0} = \frac{p_0 r_0^5}{q_0^5} \qquad \frac{q_0}{r_0} = x$$

$$x^5(x-1) = \frac{p_0}{2(\alpha-\beta)} = \frac{2.7\times 10^{10}}{2\times 2.32\times 10^{11}} = .058$$

$$x = 1.046$$

$$.046\times 3300 = 152 \text{ cm}$$

$$\frac{W}{x^2} = \text{en. in gas}$$

$$W_{el} = 4\pi(\alpha-\beta) r_0 (q_0 - r_0)^2 = 4\pi(\alpha-\beta) r_0^3 (x-1)^2$$

$$= 4\pi r_0^3 \cancel{(\alpha-\beta)} \frac{p_0^2}{4(\alpha-\beta)^{\cancel{2}} x^{10}}$$

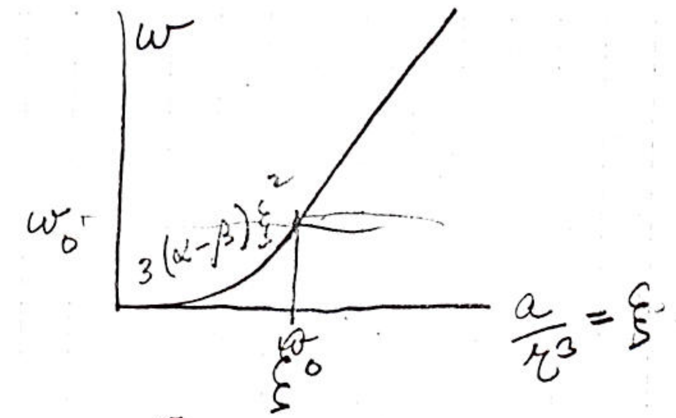
$$W = \frac{p_0 V_0}{\gamma - 1} = \frac{3}{2} p_0 V_0$$

5 % of energy is elastic radiation

period of order .040 sec

Assume plastic flow for energy density $> w_0$

En density



$$w = \begin{cases} 3(\alpha-\beta)\,\xi^2 & \text{when } < w_0 \qquad \xi < \xi_0 = \sqrt{\dfrac{w_0}{3(\alpha-\beta)}} \\ w_0 + 6\xi_0(\alpha-\beta)(\xi-\xi_0) & \end{cases}$$

$$W = 4\pi\int_{r_0}^{\infty} w\, r^2\, dr = 4\pi\int_{r_0}^{r_1}\left[w_0 + 6\xi_0(\alpha-\beta)\left(\frac{a}{r^3}-\xi_0\right)\right] r^2\, dr +$$

$$+\,4\pi\int_{r_1}^{\infty} 3(\alpha-\beta)\,\frac{a^2}{r^6}\, r^2\, dr \qquad \frac{a}{r_1^3} = \xi_0$$

$$\frac{W}{4\pi} = (\alpha-\beta)\,\frac{a^2}{r_1^3} + 3(\alpha-\beta)\left(\xi_0^2 - 2\,\xi_0^2\right)\frac{r_1^3 - r_0^3}{3} + 6(\alpha-\beta)\,\xi_0 a \ln\frac{r_1}{r_0}$$

$$\frac{W}{4\pi} = (\alpha-\beta)\,\xi_0 a + (\alpha-\beta)\left(\xi_0^2 - 2\,\xi_0^2\right)\left(\frac{a}{\xi_0} - r_0^3\right) + 2(\alpha-\beta)\,\xi_0 a \ln\frac{a}{\xi_0 r_0^3}$$

$$\frac{\partial V}{\partial a} = 4\pi \qquad \frac{\partial W}{\partial V} = p = \frac{1}{4\pi}\frac{\partial W}{\partial a}$$

$$\frac{p}{\alpha-\beta} = \xi_0 + \xi_0 - 2\xi_0 + 2\xi_0 \ln \frac{a}{\xi_0 r_0^3} + 2\xi_0$$

$$= 2\xi_0 + 2\xi_0 \ln \frac{a}{\xi_0 r_0^3}$$

$$p = \underbrace{2(\alpha-\beta)\xi_0} + 2(\alpha-\beta)\xi_0 \ln \frac{a}{\xi_0 r_0^3} \qquad \frac{r_1^3}{r_0^3} = 1.03$$

Assume $\xi_0 = .01$

$$\frac{20}{400} = .05 = .01 + .04$$

$$P = p_{rr} \qquad Q = p_{\perp\perp}$$

$$\frac{\partial}{\partial r}\left\{r^2 P(r)\right\} = 2rQ \quad (\text{for sphere})$$

$$\frac{d}{dr}\left\{r P(r)\right\} = Q \quad (\text{for cylinder})$$

In elastic case

$$P = \alpha\,(q'-1) + 2\beta\left(\frac{q}{r}-1\right)$$

$$Q = (\alpha+\beta)\left(\frac{q}{r}-1\right) + \beta\,(q'-1)$$

$\cancel{2r(\alpha+\beta)+2r\beta}$ $\cancel{2r\alpha}$ $\cancel{4r\beta}$

$$\cancel{\alpha}\, r^2\left[\alpha q'' + 2\beta\frac{q'}{r} - 2\beta\frac{q}{r^2}\right] + 2\cancel{\alpha} r\left[\alpha(q'-1) + 2\beta\left(\frac{q}{r}-1\right)\right] =$$

$$= 2r\,(\alpha+\beta)\left(\frac{q}{r}-1\right) + 2r\,\beta\,(q'-1)$$

$$q''\left[\alpha r^2\right] + q'\left[\cancel{2\beta r} + 2r\alpha\right] + q\left[\cancel{2\beta + 4\beta} - 2\alpha\right]$$

$$q'' + \frac{2q'}{r} - \frac{2q}{r^2} = 0$$

~~In plastic case:~~

Assume $P - Q < A$

In plastic flow case $P - Q = A$

$$\frac{d}{dr}\left\{r^2 P(r)\right\} = 2r\,(P-A)$$

$$P = P_0 - 2A\ln\frac{q}{q_0}$$

for cylinder without factor 2